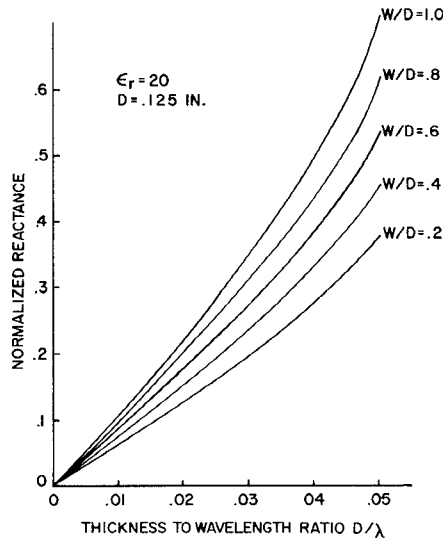
Fig. 4. A family of short-circuit-reactance curves derived from Fig. 2 for $\epsilon_r = 12$.Fig. 5. A family of short-circuit-reactance curves derived from Fig. 3 for $\epsilon_r = 20$.

III. CONCLUSIONS

It has been shown that a significant end effect exists when a slot line is shorted. The apparent position of the short is located some distance beyond the end of the slot. At a reference plane coincident with the end of the slot the termination appears as an inductive reactance which increases with both W/D and D/λ . The effect is not linear.

Experimental data have been used to generate families of curves which should prove useful for design purposes until such time as theoretical results on this effect are available. They will also be useful as a basis for comparison when a theory is developed.

ACKNOWLEDGMENT

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Slot Line with Thick Metal Coating

T. KITAZAWA, Y. FUJIKI, Y. HAYASHI, AND M. SUZUKI

Abstract—Formulas and curves are given for the phase constant of slot line with metal-coating thickness greater than zero. Change in the phase constant is about a 1-percent decrease even though metal-coating thickness is 2 percent of the slot width.

Slot line has been approximately analyzed by Cohn [1] and recently a rigorous solution was obtained by Itoh and Mittra [2]. These theories, however, neglect the effect of the metal-coating thickness. In this short paper we analyze slot line with metal-coating thickness greater than zero and evaluate this effect.

A cross section of slot line is shown in Fig. 1. In this short paper the network analytical methods of electromagnetic fields [3] are employed. First we express the transverse fields E_t , H_t in the regions $z > t$, $t > z > 0$, $0 > z > -h$, and $-h > z$ by the following Fourier integral:

1) $z > t$, $0 > z > -h$, and $-h > z$; $-\infty < x < \infty$

$$\begin{Bmatrix} E_t \\ H_t \end{Bmatrix} = \frac{1}{\sqrt{2\pi}} \sum_{l=1}^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i\beta y} \begin{Bmatrix} V_l(\alpha, \beta; z) f_l(\alpha, \beta; x) \\ I_l(\alpha, \beta; z) g_l(\alpha, \beta; x) \end{Bmatrix} d\alpha d\beta \quad (1)$$

where

$$\begin{aligned} f_1 &= \frac{j}{\sqrt{2\pi K}} (x_0 \alpha + y_0 \beta) e^{-j\alpha x}, & g_1 &= \frac{-j}{\sqrt{2\pi K}} (x_0 \beta - y_0 \alpha) e^{-j\alpha x} \\ f_2 &= \frac{j}{\sqrt{2\pi K}} (x_0 \beta - y_0 \alpha) e^{-j\alpha x}, & g_2 &= \frac{j}{\sqrt{2\pi K}} (x_0 \alpha + y_0 \beta) e^{-j\alpha x} \\ K &= \sqrt{\alpha^2 + \beta^2}. \end{aligned} \quad (2)$$

2) $t > z > 0$; $|x| \leq W/2$

$$\begin{Bmatrix} E_t \\ H_t \end{Bmatrix} = \frac{1}{\sqrt{2\pi}} \sum_{l=1}^2 \int_{-\infty}^{\infty} \sum_{n=0}^{\infty} \epsilon_l(n) e^{-i\beta y} \begin{Bmatrix} V_l(\alpha_n, \beta; z) f_l(\alpha_n, \beta; x) \\ I_l(\alpha_n, \beta; z) g_l(\alpha_n, \beta; x) \end{Bmatrix} d\beta \quad (3)$$

where

$$\begin{aligned} \epsilon_l(n) &= \begin{cases} 0 & (n=0, l=1) \\ 1/\sqrt{2} & (n=0, l=2) \\ 1 & (n \neq 0) \end{cases} \\ f_1 &= \frac{-1}{K_n} \sqrt{\frac{2}{W}} (x_0 \alpha_n \cos(\alpha_n x) - y_0 \beta \sin(\alpha_n x)) \\ g_1 &= \frac{-1}{K_n} \sqrt{\frac{2}{W}} (x_0 \beta \sin(\alpha_n x) + y_0 \alpha_n \cos(\alpha_n x)) \\ f_2 &= \frac{1}{K_n} \sqrt{\frac{2}{W}} (x_0 \beta \cos(\alpha_n x) - y_0 \alpha_n \sin(\alpha_n x)) \\ g_2 &= \frac{1}{K_n} \sqrt{\frac{2}{W}} (x_0 \alpha_n \sin(\alpha_n x) + y_0 \beta \cos(\alpha_n x)) \\ \alpha_n &= 2n\pi/W, & K_n &= \sqrt{\alpha_n^2 + \beta^2}. \end{aligned} \quad (4)$$

3) $t > z > 0$; $|x| > W/2$

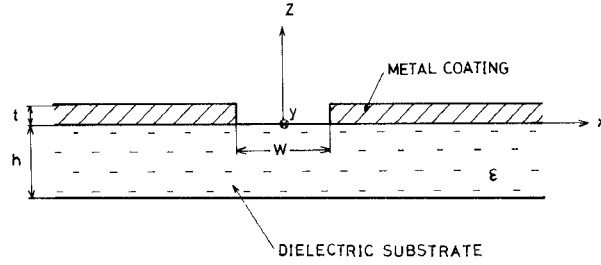
$$E_t = 0, \quad H_t = 0 \quad (5)$$

where x_0 , y_0 , and z_0 are unit vectors along the x , y , and z axis, respectively, and $l=1$ and $l=2$ represent E waves ($H_z=0$) and H waves ($E_z=0$), respectively. V_l and I_l are mode voltages and mode currents, and f_l and g_l are vector-mode functions which satisfy boundary conditions

$$E_y = 0, \quad \frac{\partial H_y}{\partial x} = 0 \quad x = \pm \frac{W}{2}, \quad 0 < z < t \quad (6)$$

and the following orthonormal properties:

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Fig. 1. Slot line with thick metal coating. ϵ is the dielectric constant.

$$\int_{-W/2}^{W/2} g_{11}^*(\alpha_n', \beta; x) \cdot z_0 \times f_1(\alpha_n, \beta; x) dx = \delta_{11'} \delta_{nn}$$

$$\int_{-\infty}^{\infty} g_{11}^*(\alpha', \beta; x) \cdot z_0 \times f_1(\alpha, \beta; x) dx = \delta_{11'} \delta(\alpha - \alpha') \quad (7)$$

$\delta_{11'}$ Kronecker's delta;
 $\delta(\alpha - \alpha')$ Dirac's δ -function

where the symbol * signifies complex conjugate function. Substituting (1)–(4) into Maxwell's field equation we obtain the following transmission-line equations:

$$-\frac{dV_l}{dz} = j a_1 I_l(z), \quad -\frac{dI_l}{dz} = j c_1 V_l(z). \quad (8)$$

a_1 and c_1 are given by

$$\begin{aligned} a_1 &= \omega \mu_0 - \xi^2 / \omega \tilde{\epsilon}, & a_2 &= \omega \mu_0 \\ c_1 &= \omega \tilde{\epsilon}, & c_2 &= \omega \tilde{\epsilon} - \xi^2 / \omega \mu_0 \end{aligned} \quad (9)$$

$$\xi = \begin{cases} K_n & (0 < z < t) \\ K & (\text{other region}) \end{cases} \quad \tilde{\epsilon} = \begin{cases} \epsilon & (-h < z < 0) \\ \epsilon_0 & (\text{other region}) \end{cases}$$

Denote by $M_1(x', y')$ and $M_2(x', y')$ the magnetic currents on the surfaces $z=0$ and $z=t$, respectively. Solving the transmission-line equations (8), considering the field continuity on boundary surfaces, and using (7), the mode currents at $z=\pm 0$ and $z=t \pm 0$ can be expressed as follows:

$$\begin{aligned} I_1(+0) &= \frac{-1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-W/2}^{W/2} \frac{j\omega\epsilon_0}{\gamma_n} g_1^*(\alpha_n, \beta; x') \cdot \tilde{M}_1(\alpha_n, \beta; x', y') e^{j\beta y'} dx' dy' \\ I_2(+0) &= \frac{-1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-W/2}^{W/2} \frac{j\gamma_n}{\omega\mu_0} g_2^*(\alpha_n, \beta; x') \cdot \tilde{M}_1(\alpha_n, \beta; x', y') e^{j\beta y'} dx' dy' \\ I_1(-0) &= \frac{-1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\omega\epsilon}{\kappa} \frac{\kappa/\epsilon_r \kappa_0 + j \tan(\kappa h)}{1 + j(\kappa/\epsilon_r \kappa_0) \tan(\kappa h)} g_1^*(\alpha, \beta; x') \\ &\quad \cdot M_1(x', y') e^{j\beta y'} dx' dy' \\ I_2(-0) &= \frac{-1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\kappa}{\omega\mu_0} \frac{\kappa_0/\kappa + j \tan(\kappa h)}{1 + j(\kappa_0/\kappa) \tan(\kappa h)} g_2^*(\alpha, \beta; x') \\ &\quad \cdot M_1(x', y') e^{j\beta y'} dx' dy' \\ I_1(t+0) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\omega\epsilon_0}{\kappa_0} g_1^*(\alpha, \beta; x') \cdot M_2(x', y') e^{j\beta y'} dx' dy' \\ I_2(t+0) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\kappa_0}{\omega\mu_0} g_2^*(\alpha, \beta; x') \cdot M_2(x', y') e^{j\beta y'} dx' dy' \\ I_1(t-0) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-W/2}^{W/2} \frac{j\omega\epsilon_0}{\gamma_n} g_1^*(\alpha_n, \beta; x') \cdot \tilde{M}_2(\alpha_n, \beta; x', y') e^{j\beta y'} dx' dy' \\ I_2(t-0) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-W/2}^{W/2} \frac{j\gamma_n}{\omega\mu_0} g_2^*(\alpha_n, \beta; x') \\ &\quad \cdot \tilde{M}_2(\alpha_n, \beta; x', y') e^{j\beta y'} dx' dy' \end{aligned} \quad (10)$$

where

$$\begin{aligned} \kappa_0 &= \sqrt{\omega^2 \epsilon_0 \mu_0 - K^2} & \kappa &= \sqrt{\omega^2 \epsilon_r \mu_0 - K^2} \\ \gamma_n &= \sqrt{\omega^2 \epsilon_0 \mu_0 - K_n^2} & \epsilon_r &= \epsilon/\epsilon_0 \\ \tilde{M}_1 &= \cot(\gamma_n t) M_1(x', y') - \csc(\gamma_n t) M_2(x', y') \end{aligned}$$

$$\tilde{M}_2 = \cot(\gamma_n t) M_2(x', y') - \csc(\gamma_n t) M_1(x', y').$$

Now we assume that M_1 and M_2 may be expressed as follows:

$$\begin{aligned} M_1(x', y') &= m_1(x') e^{-j\beta_0 y'} & (|x'| < W/2) \\ &= 0 & (\text{otherwise}) \\ M_2(x', y') &= m_2(x') e^{-j\beta_0 y'} & (|x'| < W/2) \\ &= 0 & (\text{otherwise}) \end{aligned} \quad (11)$$

where β_0 is a propagation constant. By substituting (10) and (11) into (1) and (3) we obtain the transverse fields on the surfaces $z=\pm 0$ and $z=t \pm 0$, and the determinantal equation for the propagation constant is obtained by using a continuity of magnetic fields on the boundary surfaces $z=0$ and $z=t$. The determinantal equation thus obtained involves x and y components of the vector magnetic current; however, the slot width is usually very small compared to a wavelength, so that for the lowest order hybrid mode the transverse magnetic current can be neglected. Ignoring the transverse magnetic current, the determinantal equation results in the following integral representation.

$$\begin{aligned} &\int_{-W/2}^{W/2} \int_{-\infty}^{\infty} \frac{1}{K'^2} \left\{ \frac{\omega\epsilon_0}{\kappa_0'} \alpha^2 - \frac{\kappa_0'}{\omega\mu_0} \beta_0^2 \right\} m_{2y}(x') e^{-j\alpha(x-x')} d\alpha dx' \\ &= \frac{4\pi}{W} \sum_{n=0}^{\infty} \frac{1}{K_n'^2} \left\{ \frac{\omega\epsilon_0}{\gamma_n'} \epsilon_n' \alpha_n'^2 - \frac{\gamma_n'}{\omega\mu_0} \epsilon_n'' \beta_0'^2 \right\} \left\{ \csc(\gamma_n' t) m_{1y}(x') \right. \\ &\quad \left. - \coth(\gamma_n' t) m_{2y}(x') \right\} \cos(\alpha_n x) \cos(\alpha_n x') dx' \\ &\int_{-W/2}^{W/2} \int_{-\infty}^{\infty} \frac{1}{K'^2} \left\{ \frac{\omega\epsilon_0}{\kappa_0'} \frac{1 + (\epsilon_r \kappa_0' / \kappa') \tan(\kappa' h)}{1 - (\kappa' / \epsilon_r \kappa_0') \tan(\kappa' h)} \alpha^2 \right. \\ &\quad \left. - \frac{\kappa_0'}{\omega\mu_0} \frac{1 - (\kappa' / \kappa_0') \tan(\kappa' h)}{1 + (\kappa_0' / \kappa') \tan(\kappa' h)} \beta_0'^2 \right\} m_{1y}(x') e^{-j\alpha(x-x')} d\alpha dx' \\ &= \frac{4\pi}{W} \sum_{n=0}^{\infty} \int_{-W/2}^{W/2} \frac{1}{K_n'^2} \left\{ \frac{\omega\epsilon_0}{\gamma_n'} \epsilon_n' \alpha_n'^2 - \frac{\gamma_n'}{\omega\mu_0} \epsilon_n'' \beta_0'^2 \right\} \left\{ \csc(\gamma_n' t) m_{2y}(x') \right. \\ &\quad \left. - \coth(\gamma_n' t) m_{1y}(x') \right\} \cos(\alpha_n x) \cos(\alpha_n x') dx' \end{aligned} \quad (12)$$

where

$$\begin{aligned} K' &= \sqrt{\alpha^2 + \beta_0^2} & K_n' &= \sqrt{\alpha_n'^2 + \beta_0^2} \\ \kappa_0' &= \sqrt{K'^2 - \omega^2 \epsilon_0 \mu_0} & \kappa' &= \sqrt{\omega^2 \epsilon_r \mu_0 - K'^2} \\ \gamma_n' &= \sqrt{K_n'^2 - \omega^2 \epsilon_0 \mu_0} \\ \epsilon_n' &= \begin{cases} 0 & (n=0) \\ 1 & (n \neq 0) \end{cases} & \epsilon_n'' &= \begin{cases} 1/2 & (n=0) \\ 1 & (n \neq 0) \end{cases} \end{aligned}$$

where x lies within the slot region $|x| \leq W/2$, and $m_{1y}(x')$ and $m_{2y}(x')$ are the longitudinal magnetic currents on the surfaces $z=0$ and $z=t$, respectively. The longitudinal magnetic-current distributions are unknown, so that appropriate trial functions must be selected. A first-order solution for the lowest hybrid mode may be obtained by assuming magnetic-current distribution as being equal to that obtained under quasi-TEM conditions. We assume the following distributions:

$$\begin{aligned} m_{1y}(x') &= m_{10} / \sqrt{1 - (2x'/W)^2} \\ m_{2y}(x') &= m_{20} / \sqrt{1 - (2x'/W)^2} \end{aligned} \quad (13)$$

which are the magnetic-current distributions on an infinitely thin slot in a free space, where m_{10} and m_{20} are constant values. From (12) and (13) phase constant can be obtained, for example, by assuming that

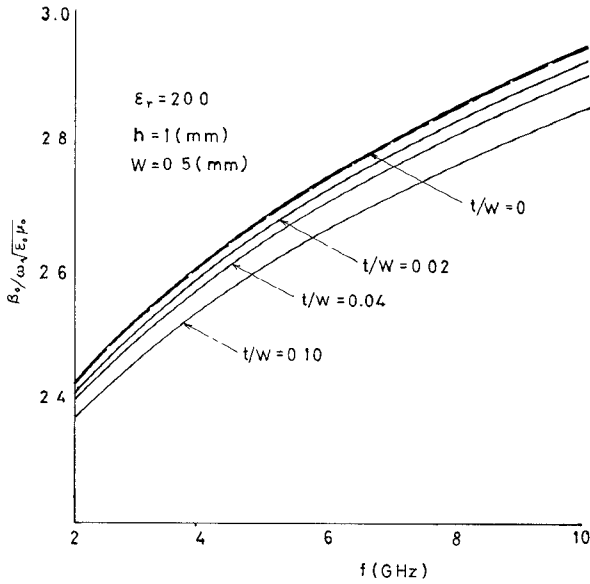


Fig. 2. Relative phase constant ($\beta/\omega\sqrt{\epsilon_r\mu_0}$). Solid line represents present method and broken line represents Cohn's results.

(12) is satisfied only at the center of the slot instead of over the slot range [4]. In this short paper we obtain the solution by applying the following numerical calculation method. We multiply the first equation of (12) by $m_{1y}(x)$ and the second equation by $m_{2y}(x)$, and integrate in the x region using (13). On eliminating m_{10} and m_{20} , we finally obtain the following determinantal equation:

$$\{G_1(\beta_0) - G_4(\beta_0)\} \{G_2(\beta_0) - G_4(\beta_0)\} = \{G_3(\beta_0)\}^2 \quad (14)$$

where

$$\begin{aligned} G_1(\beta_0) &= \int_0^\infty \frac{1}{K'^2} \left\{ \frac{\omega\epsilon_0}{\kappa_0'} \alpha^2 - \frac{\kappa_0'}{\omega\mu_0} \beta_0^2 \right\} J_0^2 \left(\frac{W}{2} \alpha \right) d\alpha \\ G_2(\beta_0) &= \int_0^\infty \frac{1}{K'^2} \left\{ \frac{\omega\epsilon_0}{\kappa_0'} \frac{1 + (\epsilon_r\kappa_0'/\kappa') \tan(\kappa'h)}{1 - (\kappa'/\epsilon_r\kappa_0') \tan(\kappa'h)} \alpha^2 \right. \\ &\quad \left. - \frac{\kappa_0'}{\omega\mu_0} \frac{1 - (\kappa'/\kappa_0') \tan(\kappa'h)}{1 + (\kappa_0'/\kappa') \tan(\kappa'h)} \beta_0^2 \right\} J_0^2 \left(\frac{W}{2} \alpha \right) d\alpha \\ G_3(\beta_0) &= \frac{\pi}{W} \frac{\gamma_0'}{\omega\mu_0} \text{csch}(\gamma_0't) - \frac{2\pi}{W} \sum_{n=1}^\infty \frac{1}{K_n'^2} \left\{ \frac{\omega\epsilon_0}{\gamma_n'} \alpha_n^2 - \frac{\gamma_n'}{\omega\mu_0} \beta_0^2 \right\} \\ &\quad \cdot \text{csch}(\gamma_n't) J_0^2 \left(\frac{W}{2} \alpha_n \right) \\ G_4(\beta_0) &= \frac{\pi}{W} \frac{\gamma_0'}{\omega\mu_0} \coth(\gamma_0't) - \frac{2\pi}{W} \sum_{n=1}^\infty \frac{1}{K_n'^2} \left\{ \frac{\omega\epsilon_0}{\gamma_n'} \alpha_n^2 - \frac{\gamma_n'}{\omega\mu_0} \beta_0^2 \right\} \\ &\quad \cdot \coth(\gamma_n't) J_0^2 \left(\frac{W}{2} \alpha_n \right) \end{aligned}$$

$J_0(x)$ zero-order Bessel function.

From (14) we can find the phase constant for the lowest hybrid mode, which is expected to be in the range $\beta_0 \geq \omega\sqrt{\epsilon_0\mu_0}$. G_3 and G_4 converge very rapidly; however, the rate of convergence of G_1 and G_2 is slow but can be improved by the following procedure. When $\alpha \rightarrow \infty$, the integrand of G_1 becomes

$$F_{1\infty} = \lim_{\alpha \rightarrow \infty} F_1 = \left(\omega\epsilon_0 - \frac{\beta_0^2}{\omega\mu_0} \right) \frac{\alpha}{\alpha^2 + \beta_0^2} J_0^2 \left(\frac{W}{2} \alpha \right). \quad (15)$$

G_1 may be rewritten as follows:

$$G_1 = \int_0^\infty (F_1 - F_{1\infty}) d\alpha + \int_0^\infty F_{1\infty} d\alpha. \quad (16)$$

The first integral on the right converges rapidly compared to G_1 , while the second may be expressed in closed form

$$\int_0^\infty F_{1\infty} d\alpha = \left(\omega\epsilon_0 - \frac{\beta_0^2}{\omega\mu_0} \right) I_0 \left(\frac{W}{2} \beta_0 \right) K_0 \left(\frac{W}{2} \beta_0 \right) \quad (17)$$

$I_0(x)$ modified Bessel function of the first kind;
 $K_0(x)$ modified Bessel function of the second kind.

Similarly for G_2

$$G_2 = \int_0^\infty (F_2 - F_{2\infty}) d\alpha + \left(\omega\epsilon_0\epsilon_r - \frac{\beta_0^2}{\omega\mu_0} \right) I_0 \left(\frac{W}{2} \beta_0 \right) K_0 \left(\frac{W}{2} \beta_0 \right). \quad (18)$$

The computed results are shown in Fig. 2 and compared with Cohn's theory [1]. The results, supposing the thickness equal to zero, are in good agreement with Cohn's results. When the t/W ratio is 0.02, the change in the phase constant is about 1 percent, and so the effect of metal-coating thickness usually can be neglected. These numerical calculations were carried out by the electronic computer FACOM 230-60. The calculation time is about 30 s per one structure.

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Some General Observations on the Tuning Characteristics of "Electromechanically" Tuned Gunn Oscillators

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Abstract—The nature of mechanical and electronic (varactor) tuning characteristics of "electromechanically" tuned Gunn oscillators in waveguide and coaxial configurations has been investigated and their interactions studied. Some general conclusions about the family of electromechanically tuned Gunn oscillators have been drawn and their limitations pointed out. It is suggested that these limitations are imposed by a distributed circuit on a point source.

INTRODUCTION

Gunn-effect oscillators are finding increasing applications as sources of microwave power. Their tunability and ease of operation has contributed to their popularity. Mechanically tuned Gunn oscillators in both coaxial and waveguide cavities have been reported in the literature [1]–[5]. The efforts were mainly devoted to a proper understanding of the tuning characteristics, explaining the mode switching observed in various cases, and the load dependence of Gunn-device performance. Electronic (varactor) tuning of Gunn oscillators has also been studied by many authors in both coaxial and waveguide configurations [6]–[8].

However, a majority of Gunn oscillators required for practical applications are of the "electromechanically" tuned type, and most operational modes demand high-speed electronic tuning about the frequency set by the mechanical tuning. Because of their high speed of operation, varactor diodes are invariably used for these applications. There is a lack of reported work in the literature on electromechanically tuned Gunn oscillators. Although intensive investigations have been made both on mechanical and electronic tuning of Gunn oscillators, the interaction between the two has been studied in a rather piecemeal way. The general approach taken by most workers is to optimize the performance in a restricted (mechanical) frequency range [9]. Such an approach fails to demonstrate the difficulties encountered when trying to broad-band these oscillators.

An attempt has been made here to present the tuning characteristics of electromechanically tuned Gunn oscillators in a wider perspective. In the broad frequency range in which an understanding of the tuning characteristics of electromechanically tuned Gunn oscillators is being sought, an equivalent circuit representation of the oscillator is very difficult to obtain because of the complexity of the microwave circuit involved, most elements of which are frequency